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FLOW IN OPEN CHANNELS

by Edward F. Wilsey

HYDRAULICS DIVISION

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PAPERS

FLOW IN OPEN CHANNELS

By Edward F. Wilsey,[1] A. M. ASCE

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Synopsis

In 1775 Chezy suggested his formula for the velocity of flow in open channels in terms of the hydraulic radius and the slope of the channel. In this paper the Chezy formula is derived in a dimensional form—the alternate stages of flow are analyzed to illustrate the division of open channel flow into two categories. The dimensional equation for open channels is developed on the basis of experimental data. The stream function, used extensively in aerodynamics, is introduced as a possible explanation for the entrainment of air in steep chutes, and a formula is developed for the velocity of flow in channels set at a gradual slope.

Notation

The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in the Appendix.

The Fundamental Equation

Engineers are acquainted with the formula for the drag created by the flow of a fluid along a flat plate in which two surfaces offer resistance to the flow. An open channel can be conceived as a flat plate warped into a cylinder. An unclosed cylinder forms an open channel; a closed cylinder forms a pipe. A fluid flowing in the open cylinder will create a drag on the inside surface. The drag formula can therefore be applied to both open channel flow or pipe flow. When applied to chutes, the formula results in a dimensional equation; when applied to pipes, the formula yields the Darcy and Bernoulli equations for uniform flow. For chutes,

$$4\, C_d = f \qquad (1)$$

in which C_d is the coefficient of drag and f denotes the friction factor in Darcy's formula. The validity of applying the drag formula for uniform open channel flow is thus demonstrated. Fig. 1 shows a free-body diagram of the flow of a fluid down a chute having a uniform cross section. The depth and average

velocity are constant and the forces on the ends of the section caused by the pressure within the fluid are balanced. Since the flow has uniform velocity, the forces acting upon the free-body are in equilibrium. Equating the components parallel to the chute results in

$$W \sin \theta = d \qquad (2a)$$

or

$$\gamma\, A\, L \sin \theta = C_d \frac{\rho}{2} P\, L\, V^2 \qquad (2b)$$

in which W is the weight of the water in the section under consideration, θ represents the angle at which the chute is set, d is the drag force, γ is the specific weight, A denotes the cross-sectional area, L is the length of the section, ρ denotes the density, P is the wetted perimeter, and V represents the mean velocity of flow.

However, since $A/P = R$ and $\gamma/\rho = g$, Eq. 2b can be rearranged to yield

$$V = \sqrt{\frac{2\, g}{C_d} R \sin \theta} \qquad (2c)$$

in which R is the hydraulic radius and g denotes the gravitational acceleration.

Eq. 2c is a dimensional equation and is used in the form,

$$V = B \sqrt{2\, g\, R \sin \theta} \qquad (2d)$$

in which

$$B = \sqrt{\frac{1}{C_d}} \qquad (2e)$$

Two Types of Flow in Open Channels

For ideal steady flow there are two depths with the same specific energy, E_s. The specific energy at any point is

$$E_s = \frac{V^2}{2\, g} + y \qquad (3a)$$

in which y denotes depth. For the two depths,

$$y_u + \frac{V^2_u}{2\, g} = y_l + \frac{V^2_l}{2\, g} \qquad (3b)$$

in which the subscripts u and l indicate the upper and lower stages, respectively.

If the discharge per unit width of rectangular cross section of flow is q, the equation of continuity becomes

$$y_u\, V_u = y_l\, V_l = q \qquad (4a)$$

or

$$V_u = \frac{q}{y_u} \qquad (4b)$$

and

$$V_l = \frac{q}{y_l} \qquad (4c)$$

Eq. 3*b* then becomes

$$y_u - y_l = \frac{q^2}{2\,g}\left(\frac{y^2{}_u - y^2{}_l}{y^2{}_u\, y^2{}_l}\right) \qquad (5a)$$

from which

$$y^2{}_u\, y^2{}_l = (y_u + y_l)\,\frac{q^2}{2\,g} \qquad (5b)$$

For a given discharge there is always a depth at which the specific energy is a minimum. Thus, from

$$E_s = y + \frac{q^2}{2\,g\,y^2} \qquad (6a)$$

there results for a minimum value of E_s

$$\frac{d}{d\,y}\,(E_s) = 1 - \frac{2\,q^2}{2\,g\,y^3} = 0 \qquad (6b)$$

This depth for minimum energy is called the critical depth y_c, and from Eq. 6*b* is expressed by

$$y_c = \sqrt[3]{\frac{q^2}{g}} \qquad (7a)$$

or

$$y^3{}_c = \frac{q^2}{g} \qquad (7b)$$

Substituting Eq. 7*b* into Eq. 5*b*,

$$2\,y^2{}_u\, y^2{}_l = (y_u + y_l)\, y^3{}_c \qquad (8)$$

Eq. 5*b* shows that any device that will convert flow from one stage to another will be a satisfactory measuring device if the loss of energy between the two sections is negligible. If the depths at the upper and lower stages are measured, the discharge Q can be computed from the following equation which is developed from Eq. 5*b*:

$$Q = \sqrt{2\,g}\;\frac{b\,y_u\,y_l}{\sqrt{y_u + y_l}} \qquad (9)$$

in which b is the width of the channel. Eq. 9 is rational, dimensional, and includes the head of the velocity of approach. The upper and lower stages are easy to measure, and it should be possible to design more than one device that will produce both stages.

It has been demonstrated that flow above the critical depth is tranquil and the stage below the critical depth rapid or shooting. The critical depth can be chosen as the criterion for the choice of open-channel formulas. For an average depth above the critical depth, low-velocity formulas must be used; for a depth below the critical depth, steep-slope formulas must be used. If the average depth is near the critical depth, unstable flow is likely to occur.

The Dimensional Analysis.—For a dimensional analysis an equation must be found for the velocity in an open channel—in terms of the density and dynamic viscosity of the fluid, the dimensions of the open channel, and the

acceleration of gravity. Because the kinematic viscosity ν is equal to the ratio of the dynamic viscosity μ to the density ρ, the velocity V can be expressed as a function of $1/\nu$, g, and J a linear dimension of the channel.

Using Taylor's expansion,

$$V = \Sigma \left(\frac{1}{\nu}\right)^i J^j g^k \qquad (10a)$$

However $(1/\nu)^i J^j g^k$ is dimensionally equal to

$$\left[\frac{\text{time}}{(\text{length})^2}\right]^i (\text{length})^j \left[\frac{\text{length}}{(\text{time})^2}\right]^k \qquad (10b)$$

from which

$$\frac{\text{length}}{\text{time}} = (\text{length})^{-2i+j+k} (\text{time})^{i-2k} \qquad (11a)$$

Eq. 11*a* results in

$$1 = -2i + j + k \qquad (11b)$$

$$-1 = i - 2k \qquad (11c)$$

Solving for j and k in terms of i,

$$j = \tfrac{1}{2} + \tfrac{3}{2} i \qquad (11d)$$

$$k = \tfrac{1}{2} + \tfrac{1}{2} i \qquad (11e)$$

Eq. 10*a* becomes

$$V = \Sigma \left(\frac{1}{\nu}\right)^i J^{\frac{1}{2}+\frac{3}{2}i} g^{\frac{1}{2}+\frac{1}{2}i} \qquad (12a)$$

from which

$$V = \sqrt{g J}\, \Sigma \left(\frac{\sqrt{g J^3}}{\nu}\right)^i \qquad (12b)$$

which can be rewritten as

$$V = \sqrt{g J}\, \Phi \left(\frac{\sqrt{g J^3}}{\nu}\right) \qquad (12c)$$

Comparing Eq. 12*c* with Eq. 2*d*, it can be seen that the term J under the second radical is equivalent to R, and the factors $\sin \theta$ and 2 must be introduced to make this part of Eq. 12*c* agree with Eq. 2*d*. Thus,

$$V = \sqrt{2 g R \sin \theta}\, \Phi \left(\frac{\sqrt{g J^3}}{\nu}\right) \qquad (13)$$

From an examination of Eqs. 2*d* and 13 it is seen that

$$B = \Phi \left(\frac{\sqrt{g J^3}}{\nu}\right) = \Phi \left(\frac{\rho \sqrt{g J^3}}{\mu}\right) \qquad (14a)$$

The dimensionless number B is not easy to interpret. The density and the dynamic viscosity are well defined, but the meaning of the term J is obscure.

On the assumption that J represents the hydraulic radius R,

$$B = \Phi\left(\frac{\sqrt{g\,R^3}}{\nu}\right) \qquad (14b)$$

It is usual to represent the Φ-function in the following form:

$$B = C\left(\frac{\sqrt{g\,R^3}}{\nu}\right)^n \qquad (14c)$$

If n is equal to $\frac{1}{9}$ and ρ and μ are assumed to be constant, there results Manning's coefficient for Chezy's formula—that is,

$$B = C_m\,R^{1/6} \qquad (14d)$$

This interpretation has been acceptable since 1890, when Robert Manning introduced his formula.

If the critical depth is introduced for the J-factor in the Φ-function, then

$$B = \Phi\left(\frac{\sqrt{g\,y^3_c}}{\nu}\right) = \Phi\left[\frac{\frac{Q}{b}}{\nu}\right] = \Phi\left(\frac{V\,y}{\nu}\right) \qquad (15)$$

Thus in Eq. 15, B is a function of the Reynolds number R. This may have some bearing on that flow whose depth is above the critical depth. However, for shooting flow, if R is plotted (on logarithmic paper) against B (the resistance coefficient) a series of horizontal lines result, showing that for a given slope,

$$B = M\left[\frac{\frac{Q}{b}}{\nu}\right]^0 = \text{a constant} \qquad (16)$$

B is therefore independent of the Reynolds number for channels set at steep slopes. This may not be true for tranquil flow.

The Experimental Evidence

Table 1 shows the results of tests performed by R. Ehrenberger on a small wooden chute 0.82 ft wide.[2] Table 2 presents the results of tests performed on

[2] "Flow in Steep Chutes with Special Reference to Self-Aeration," *Proceedings*, ASCE, Abridged Translations of Hydraulics Papers, September, 1943, p. 31.

the Kittitas wasteway near Cle Elum, Wash.

For a given slope, it can be seen that P_w, the water portion of the aerated flow, as determined by the relation,

$$P_w = \frac{Q_s}{Q_a} \qquad (17)$$

in which Q_s is the discharge before aeration and Q_a is the discharge of the aerated flow, is practically constant. It is also evident that B, the resistance coefficient in Eq. 2d, is sensibly constant for a given slope. The average value of the ratio P_w/B was computed for each slope of the wooden chute. Table 3

TABLE 1.—FLOW ON A WOODEN CHUTE

Hydraulic radius R, in feet	Measured velocity of flow, in feet per second	Water portion, P_w	Density ρ, in slugs per cubic foot	Resistance coefficient, B	$\frac{P_w}{B}$	Computed velocity of flow, in feet per second
			$\theta = 8° - 48'$			
0.050	9.61	0.790	1.53	13.9	0.0569	9.5
0.074	11.94	0.790	1.53	14.1	0.0562	11.2
0.094	13.29	0.807	1.56	14.3	0.0564	12.7
0.116	14.53	0.801	1.55	13.7	0.0575	15.6
			$\theta = 11° - 39'$			
0.048	10.24	0.770	1.49	12.7	0.0608	10.2
0.072	13.06	0.750	1.45	13.2	0.0568	12.5
0.093	14.70	0.750	1.45	13.1	0.0572	14.2
0.113	16.04	0.758	1.47	13.0	0.0584	15.7
			$\theta = 17° - 46'$			
0.045	11.48	0.729	1.41	12.2	0.0597	11.8
0.068	14.47	0.716	1.39	12.5	0.0572	14.5
0.090	16.31	0.704	1.36	12.3	0.0572	16.7
0.111	17.55	0.707	1.37	11.9	0.0594	18.5
			$\theta = 26° - 14'$			
0.043	13.22	0.675	1.30	14.9	0.0568	13.4
0.066	16.80	0.644	1.24	12.2	0.0528	16.6
0.088	18.73	0.631	1.22	11.8	0.0534	19.1
0.109	20.18	0.635	1.23	11.3	0.0562	21.3
			$\theta = 41° - 46'$			
0.047	15.78	0.513	0.990	11.5	0.0447	15.6
0.071	19.46	0.512	0.990	11.5	0.0457	19.2
0.092	21.85	0.510	0.985	11.4	0.0448	21.8
0.113	23.62	0.513	0.990	11.2	0.0458	23.2

Table 2.—Flow on a Concrete Chute

Hydraulic radius R, in feet	Measured velocity of flow, in feet per second	Water portion, P_w	Density ρ, in slugs per cubic foot	Resistance coefficient, B	$\frac{P_w}{B}$	Computed velocity of flow, in feet per second
			$\theta = 11° - 12'$			
0.44	35.4	0.64	1.24	14.5	0.044	35.2
0.45	34.0	0.66	1.28	14.9	0.044	35.7
0.68	37.8	0.80	1.55	13.6	0.059	44.0
0.68	37.0	0.81	1.57	14.3	0.057	44.0
0.70	37.8	0.78	1.51	13.3	0.059	44.4
0.69	39.1	0.76	1.47	13.9	0.055	44.0
0.96	49.8	0.73	1.41	14.9	0.048	52.0
0.96	52.5	0.69	1.34	15.7	0.044	52.0
0.96	49.5	0.74	1.43	14.8	0.050	52.0
0.96	61.3	0.60	1.16	17.8	0.034	52.0
0.98	46.6	0.86	1.66	13.9	0.062	52.6
0.98	52.2	0.77	1.49	15.5	0.050	52.6
1.14	52.2	0.76	1.47	14.4	0.053	56.6
1.13	52.0	0.77	1.49	14.4	0.053	56.4
1.14	55.4	0.72	1.39	15.3	0.047	56.6
1.15	57.4	0.69	1.34	15.8	0.043	56.9
1.26	55.2	0.74	1.43	14.5	0.051	59.5
1.26	56.5	0.73	1.41	14.8	0.049	59.5
1.26	58.6	0.70	1.35	15.4	0.045	59.5
1.26	64.3	0.75	1.45	16.9	0.044	59.5
1.36	57.8	0.77	1.49	14.7	0.052	61.8
1.37	57.0	0.78	1.51	14.4	0.054	62.1
1.38	61.1	0.72	1.39	15.5	0.047	62.3
1.39	66.8	0.65	1.26	15.3	0.043	62.5
1.45	64.6	0.67	1.30	15.9	0.042	63.8
1.45	63.2	0.70	1.35	15.5	0.045	63.8
1.44	63.1	0.70	1.35	15.5	0.045	63.6
1.46	63.7	0.68	1.31	15.5	0.043	64.0
1.56	66.8	0.69	1.34	15.8	0.044	66.3
1.55	66.7	0.70	1.35	15.8	0.044	66.0
1.55	63.2	0.74	1.43	15.1	0.049	66.0
1.56	67.1	0.69	1.34	14.9	0.046	66.2
1.63	73.2	0.64	1.24	16.9	0.038	67.8
1.62	67.4	0.71	1.37	15.6	0.046	67.5
1.62	69.3	0.69	1.34	14.9	0.046	67.5
1.62	65.9	0.68	1.31	15.5	0.044	67.5
			$\theta = 33° - 10'$			
0.83	54.6	0.44	0.85	10.1	0.044	63.3
0.90	66.1	0.32	0.62	11.7	0.027	66.0
0.90	67.0	0.32	0.62	12.1	0.026	66.0
1.11	73.3	0.42	0.81	11.7	0.036	73.1
1.11	71.0	0.43	0.83	11.3	0.038	73.1
1.17	70.1	0.40	0.77	11.0	0.036	74.1
1.17	65.4	0.40	0.77	10.2	0.039	74.1
1.11	74.4	0.46	0.89	11.8	0.039	73.1
1.12	77.4	0.42	0.81	12.3	0.034	73.5
1.32	73.2	0.44	0.85	10.7	0.041	79.9
1.32	79.1	0.41	0.79	11.5	0.036	79.9
1.32	77.0	0.44	0.87	11.4	0.039	79.9
1.30	81.6	0.40	0.77	12.0	0.033	79.1
1.30	76.3	0.43	0.83	11.2	0.038	79.1
1.37	79.3	0.46	0.89	11.4	0.040	81.5
1.37	82.8	0.44	0.85	11.9	0.037	81.5
1.37	83.8	0.43	0.83	11.9	0.036	81.5
1.43	92.0	0.45	0.87	12.9	0.034	83.1
1.43	82.2	0.50	0.96	11.9	0.044	83.1
1.43	87.1	0.47	0.91	12.2	0.039	83.1
1.50	81.7	0.51	0.98	12.2	0.042	85.0
1.60	90.8	0.49	0.95	12.0	0.041	87.9
1.60	87.5	0.51	0.98	11.6	0.044	87.9
1.61	93.0	0.47	0.91	12.3	0.038	88.1
1.61	85.1	0.51	0.98	11.3	0.045	88.1
1.71	92.0	0.47	0.91	11.8	0.040	91.0

TABLE 3.—RELATION BETWEEN SLOPE, WATER PORTION, AND RESISTANCE COEFFICIENT FOR A WOODEN CHUTE

Angle of inclination θ, in degrees	Cos θ	Average value of $\frac{P_w}{B}$	$\frac{P_w}{B \cos \theta}$	0.06 cos θ
8°48′	0.988	0.0568	0.0577	0.059
11°39′	0.976	0.0584	0.0598	0.059
17°46′	0.952	0.0584	0.0613	0.057
26°14′	0.897	0.0548	0.0611	0.054
41°46′	0.745	0.0453	0.0609	0.045

shows that these average values fit the cosine function of the angle of inclination of the chute. From Table 3 it can be seen that the average value of $\frac{P_w}{B \cos \theta}$ is 0.06, and thus for angles of inclination of between 8° and 42°

$$P_w = 0.06\, B \cos \theta \quad (18)$$

Introducing the value of B from Eq. 18 into Eq. 2*d* results in

$$V = 16.7\, P_w \sec \theta \sqrt{2\, g\, R \sin \theta} \quad (19)$$

for values of θ between 8° and 42°. The velocity of flow shown in Table 1 was computed from Eq. 19.

The same procedure as was used for the development of Table 3 results in Table 4 for a concrete chute.

TABLE 4.—RELATION BETWEEN SLOPE, WATER PORTION, AND RESISTANCE COEFFICIENT FOR A CONCRETE CHUTE

Angle of inclination θ, in degrees	Cos θ	Average value of $\frac{P_w}{B}$	$\frac{P_w}{B \cos \theta}$	0.0466 cos θ
11°12′	0.981	0.0458	0.0467	0.0457
33°10′	0.837	0.0390	0.0466	0.0390

From Table 4, the average value of $\frac{P_w}{B \cos \theta}$ is 0.0466 and thus for the angles of inclination between 11° and 33°,

$$P_w = 0.0466\, B \cos \theta \quad (20)$$

from which

$$V = 21.4\, P_w \sec \theta \sqrt{2\, g\, R \sin \theta} \quad (21)$$

The velocity of flow shown in Table 2 was computed from Eq. 21.

In order to find a working relationship between P_w and θ, Fig. 2 was plotted. Fig. 2 shows that P_w varies linearly with θ within the range of values covered by the tests.

The equations for these lines are as follows: For wooden chutes inclined at angles between 8° and 42°,

$$P_w = 0.875 - 0.00862\, \theta \quad (22a)$$

For concrete chutes inclined at angles between 11° and 33°,

$$P_w = 0.843 - 0.0114\,\theta \qquad (22b)$$

From Eqs. 19, 21, and 22 it can be seen that the roughness increases the numerical constant in the velocity equations and also increases the slope of the lines in Fig. 2.

The ratio of the numerical constants in Eqs. 19 and 21 is

$$\frac{21.4}{16.7} = 1.28 \qquad (23a)$$

and the ratio of the slopes of the straight lines in Fig. 2 is

$$\frac{0.0114}{0.00862} = 1.32 \qquad (23b)$$

These are interesting results, but further experimental investigation is necessary to establish a definite relationship between Eqs. 23.

Some measurements were made (by the Bureau of Reclamation, United States Department of the Interior, at Denver, Colo., in 1948) on chutes set at lesser angles of inclination than those shown in Tables 1 and 2.

TABLE 5.—FLOW CHARACTERISTICS ON A CONCRETE CHUTE

Angle of inclination θ, in degrees	Discharge Q, in cubic feet per second	Hydraulic radius R, in feet	Water portion, P_w	Computed water portion, P_w	Velocity of flow, V	Computed velocity of flow, V
4°42′	27.5	0.26	78	79	22.0	19.8
8°41′	23.3	0.21	90	74	19.6	22.8
9°00′	22.4	0.22	63	74	26.3	24.0

Table 5 indicates that the line for concrete chutes shown in Fig. 2 is valid for angles as small as 5°. For wooden chutes, however, Table 6 indicates that

TABLE 6.—FLOW CHARACTERISTICS ON A WOODEN CHUTE

Angle of inclination θ, in degrees	Discharge Q, in cubic feet per second	Hydraulic radius, R	Water portion, P_w	Computed water portion, P_w	Velocity of flow, V
1°26′	6.16	0.155	95	86	9.25
2°59′	1.24	0.107	99	85	9.82
7°7′	95.0	0.48	86	81	32.3

the line in Fig. 2 turns up toward unity at small angles. It is possible that P_w can be represented by a single mathematical expression for all angles, but more extensive data are required to accomplish this.

NATURAL BRAKING

It is apparent that the resistance coefficient largely depends on the amount of air entrained in the flow and that as the water portion of the flow decreases,

the air portion $(1 - P_v)$ increases. This phenomenon can be referred to as natural braking, which effectually avoids the destructive result of high velocity flow. Natural braking was apparent during the Kittitas tests. On the last day of the tests, approximately 1,000 cu ft per sec was flowing down the wasteway and on the steeper slope the velocity was 90 ft per sec. With this flow and slope, the air portion exceeded the water portion. Spray covered the platforms erected across the chute for point gage readings of depth; no attempt could be made to find the water surface, except at the last station, where the conditions for observation were better. Steeper slopes long enough for uniform flow to be established on spillways should present an awesome sight.

Initial Air Entrainment.—Fig. 3 shows the measured depth for the Kittitas tests and the critical depth in relation to the unit discharge down the chute. All the measured depths, despite the large air portion, are below the curve for critical depth and therefore in the shooting stage. In a similar manner every depth measured by Mr. Ehrenberger can be shown to be below the critical depth. Some experimenters believe that aeration begins at a certain velocity, but it is probable that entrainment of air begins at any depth below the critical depth. This capacity for air entrainment is a distinguishing feature of rapid flow, thus requiring (for rapid flow) the application of new formulas.

Fig. 3 shows that the depth measurement for a discharge of 11.1 cu ft per sec per ft on the steeper slope of the Kittitas wasteway is missing. No depth could be measured at this discharge because the flow assumed a pattern which can best be described as a series of surges unevenly spaced and traveling faster than the slower, shallower flow near the bottom of the chute. The vertical curve connecting the two slopes might have caused this unstable flow. Even though there was at least a 1 ft difference between the measured depth and the critical depth, unstable flow resulted. Thus, unstable flow is likely to be found at any depth near the critical depth, especially when some disturbing influence is present.

The Stream Function

Aerodynamics can be used to develop a concept to explain the mechanics of air entrainment. The essential difference between the drag on a flat plate and the drag on the walls and bottom of an open channel is that in an open channel the boundary layer cannot increase its thickness indefinitely. This is because the boundary layer from the two sides and the bottom meet, the surface becomes rough, and air entrainment is at its maximum. The boundary layer can be conceived to be a series of eddies combined with translation down the chute. If these eddies conform to the definition of circulatory flow, the circulation may be written as

$$\psi = - v'_t \, r'_t \log_e \left(\frac{r}{a}\right) \qquad (24)$$

in which ψ is the stream function, v'_t is the local circumferential velocity at distance r'_t from the center of the eddy, r denotes the radius vector of polar coordinates, and a is the radius of the outer boundary of the eddy. If the stream function for rectilinear motion down the chute is added to this stream

function, there results

$$\psi = V\, r \sin\phi - v'_t\, r'_t \log_e\left(\frac{r}{a}\right) \qquad (25)$$

Therefore, the radial and circumferential components of the velocity at any point are, respectively,

$$h' = \frac{1}{r}\frac{\partial\psi}{\partial\phi} = \frac{1}{r}\,(r\, V \cos\phi) \qquad (26a)$$

and

$$v' = -\frac{\partial\psi}{\partial r} = -\,V \sin\phi + \frac{v'_t\, r'_t}{r} \qquad (26b)$$

The total velocity along a streamline is

$$V_t = \sqrt{(h')^2 + (v')^2} = \sqrt{V^2 + \left(\frac{v'_t\, r'_t}{r}\right)^2 - 2\,\frac{V\, v'_t\, r'_t}{r}\sin\phi} \qquad (27)$$

Bernoulli's equation along a streamline in an ideal fluid is

$$p = \tfrac{1}{2}\,\rho\, V^2_t = \text{constant} = K\, p_a \qquad (28a)$$

in which p_a is the atmospheric pressure assumed constant and K is a constant. Introducting the total velocity into Bernoulli's equation results in

$$p = K\, p_a - \frac{\rho}{2}\left[V^2 + \left(\frac{v'_t\, r'_t}{r}\right)^2 - \frac{2\, V\, r'_t\, v'_t}{r}\sin\phi\right] \qquad (28b)$$

From Eq. 28*b* it can be seen that an eddy at the surface will draw in air at its center ($r = 0$).

Along the radius vector ($\phi = 0$) air will be entrained when

$$\frac{\rho}{2}\left[V^2 + \left(\frac{v'_t\, r'_t}{r}\right)^2\right] > K\, p_a \qquad (29)$$

In circulatory flow the product $v'_t\, r'_t$ is a constant and is equal to the intensity of circulation. Consequently, a fluid's ability to entrain air depends on the density of the fluid, the intensity of the circulation, the average velocity down the chute, and the value of r.

Flow in Open Channels of Low Slope

The study of flow through pipes and the study of airfoils leads to the realization that friction between a moving fluid and a surface is a function of the Reynolds number. Because the introduction into the dimensional equation of the critical depth in the term denoting the frictional resistance results in the Reynolds number, it is reasonable to use the critical depth instead of the hydraulic radius as the frictional resistance parameter in low-velocity open channel flow.

The data selected for the study of open channel flow are the measurements made by F. C. Scobey on a wide variety of channels.[3] All tests showing an

[3] "Handbook of Hydraulics," by H. W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1918, p. 404.

average depth less than the critical depth have been eliminated. If the channel did not have a rectangular cross section, the critical depth was computed from

$$y_c \doteq \sqrt[3]{\frac{\left(\frac{Q}{b}\right)^2}{g}} \quad (30)$$

since

$$\frac{Q}{b} = V\frac{A}{b} \quad (31)$$

The unit discharge is therefore computed by using the width of the channel at the surface. In considering aerated flow, the discharge of air-free water and the velocity of the flow must be measured separately.

Since no temperature measurements were included in Mr. Scobey's data, the dimensional equation was reduced to

$$V = K\, d^m{}_c \sqrt{R\, S_0} \quad (32)$$

in which S_0 is the slope of the channel. By trying values of m equal to 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, an exponent was selected which seemed to offer the best fit to the data. The following equations were adopted:

$$V = \frac{1.44}{n} d^{0.2}{}_c \sqrt{R\, S_0} \quad (33a)$$

$$V = \frac{1.255}{n}\left(\frac{Q}{b}\right)^{2/15} \sqrt{R\, S_0} \quad (33b)$$

and

$$V = \left(\frac{1.255}{n}\, y^{2/15}{}_a \sqrt{R\, S_0}\right)^{15/13} \quad (33c)$$

Table 7 shows the value of n determined from Eqs. 33 by use of the data obtained by Mr. Scobey. Also listed, for comparison, are the n-values obtained from Manning's formula.

Examination of Table 7 reveals that the wooden flumes showed the smallest variation of the friction coefficient. This is shown by the fact that 94% of the tests on wooden flumes resulted in n-values of between 0.010 and 0.015, with the greatest frequency of values occurring between 0.012 and 0.014. The wooden flume described as coated with "slime" had a coefficient of 0.016, the flume having a "sand and moss" coating had a coefficient of 0.018, and the "asphalted" flume had a coefficient of 0.020.

The tests on concrete flumes showed 55% of the coefficients to have values between 0.013 and 0.017. Eight flumes had lesser coefficients, whereas highest coefficients were produced by "moss" (0.018), "sand deposits" (0.019 and 0.021), "rough" (0.018), and "rough and broken" (0.020). The roughest coefficient (0.021) was for a "small ditch with a cement wash."

Earth channels presented a wide variation of coefficients. Channels in loam, clay, or with fine silt bottoms, however, were rather consistently smooth (n-values of between 0.013 and 0.020). Sand and gravel bottoms were rougher (0.018 to 0.036). The effect of grass and vegetation is hard to determine. The

Table 7.—Comparison of Coefficients of Friction

Measured velocity, in feet per second	Hydraulic radius, in feet	Slope, in feet per foot	Discharge, in cubic feet per second	Width at surface, in feet	Coefficient of roughness from Eqs. 33	Manning's coefficient of roughness
Concrete Flumes						
1.07	0.62	0.000584	3.02	2.7	0.021	0.024
1.38	0.52	0.000694	2.86	2.6	0.012	0.019
1.67	1.22	0.000321	27.16	11.5	0.013	0.018
1.71	0.58	0.00118	5.71	5.4	0.016	0.021
1.79	1.60	0.000482	47.83	12.9	0.021	0.025
1.88	1.02	0.000851	23.64	11.0	0.020	0.024
2.04	0.91	0.00102	25.74	12.4	0.017	0.021
2.10	0.88	0.00095	25.74	12.4	0.017	0.020
2.27	1.63	0.000574	84.32	21.0	0.018	0.022
2.45	1.30	0.000329	50.49	15.5	0.012	0.013
2.55	1.25	0.000450	54.64	16.5	0.013	0.014
2.83	0.95	0.00145	18.54	4.1	0.018	0.019
2.85	1.63	0.000629	74.86	12.7	0.016	0.018
2.87	1.62	0.000729	74.86	12.7	0.017	0.019
2.89	0.98	0.00144	18.54	4.1	0.019	0.019
2.94	1.94	0.000525	98.30	12.9	0.018	0.018
3.02	1.90	0.000639	98.30	12.9	0.017	0.019
3.34	1.50	0.000619	120.90	22.8	0.013	0.015
3.58	1.38	0.001157	114.80	21.0	0.016	0.018
3.65	2.54	0.000251	221.20	22.0	0.011	0.012
3.65	2.54	0.000284	221.20	22.0	0.011	0.013
3.74	0.84	0.000238	18.71	3.5	0.017	0.016
3.81	2.45	0.000333	469.70	49.1	0.013	0.013
3.82	2.34	0.000413	364.50	27.8	0.013	0.014
3.86	1.94	0.00062	107.60	7.0	0.015	0.015
3.94	2.07	0.000629	212.00	23.0	0.014	0.015
4.14	2.91	0.000283	316.70	23.6	0.011	0.012
4.15	2.81	0.000237	293.62	22.8	0.010	0.011
4.68	3.88	0.000388	1027.37	53.7	0.014	0.016
4.71	1.41	0.00151	111.30	14.7	0.015	0.016
4.74	1.37	0.00082	74.64	4.5	0.012	0.011
5.01	1.31	0.00126	70.60	4.5	0.013	0.013
5.48	5.23	0.000161	1824.00	59.0	0.010	0.010
Wooden Flumes						
1.08	1.94	0.0000568	43.05	13.7	0.013	0.017
1.22	2.30	0.000084	67.25	13.0	0.016	0.019
1.30	0.66	0.000362	6.97	5.9	0.013	0.016
1.54	0.93	0.000274	16.01	9.7	0.013	0.015
1.56	0.86	0.000623	15.72	9.8	0.018	0.021
1.60	1.04	0.000325	19.35	6.5	0.015	0.017
1.64	0.40	0.000868	159.34	5.9	0.013	0.014
2.08	1.96	0.000133	66.50	9.6	0.012	0.013
2.32	1.92	0.00016	92.80	15.8	0.011	0.013
2.60	1.80	0.000225	95.50	15.7	0.011	0.013
2.67	2.24	0.00020	142.10	17.7	0.012	0.014
3.06	1.62	0.00053	65.20	7.6	0.015	0.016
3.13	1.64	0.00053	69.07	7.8	0.014	0.015
3.23	1.94	0.00024	142.10	17.7	0.010	0.011
3.34	1.08	0.000104	64.02	11.9	0.014	0.015
3.34	1.08	0.000105	64.02	11.9	0.014	0.015
3.44	1.06	0.00105	64.02	11.9	0.014	0.014
3.58	0.72	0.00392	23.64	7.4	0.020	0.021
3.61	1.00	0.00148	50.31	10.0	0.015	0.016
3.71	0.99	0.00104	64.02	11.9	0.012	0.013
3.80	1.11	0.000710	63.30	12.0	0.010	0.011
3.99	0.93	0.000202	50.31	10.0	0.015	0.016
4.08	0.90	0.00193	50.31	10.0	0.014	0.015
4.35	0.87	0.00214	50.31	10.0	0.014	0.015
4.49	2.28	0.000311	251.80	9.6	0.011	0.010
5.08	1.66	0.000713	139.50	12.0	0.011	0.011
5.45	1.23	0.00213	88.74	10.0	0.014	0.014
5.79	1.35	0.00129	86.99	6.5	0.012	0.011
5.81	2.12	0.000858	228.80	12.0	0.012	0.012
5.96	1.96	0.000965	209.30	12.0	0.012	0.012
6.21	1.66	0.00169	159.34	5.9	0.015	0.014
7.30	1.49	0.00213	153.40	10.0	0.012	0.012
7.80	1.47	0.00213	160.79	10.0	0.013	0.011
7.93	1.72	0.00213	206.88	10.0	0.013	0.012
9.12	1.90	0.00210	399.77	16.0	0.012	0.011

TABLE 7.—COMPARISON OF COEFFICIENTS OF FRICTION—(*Continued*)

Measured velocity, in feet per second	Hydraulic radius, in feet	Slope, in feet per foot	Discharge, in cubic feet per second	Width at surface, in feet	Coefficient of roughness from Eqs. 33	Manning's coefficient of roughness
MASONRY-LINED FLUMES						
1.49	1.23	0.000471	19.44	7.4	0.023	0.025
1.76	1.78	0.00636	45.80	11.5	0.028	0.031
2.26	1.05	0.00160	19.44	5.1	0.027	0.027
3.82	0.86	0.001367	19.44	4.2	0.014	0.013
6.17	0.86	0.005575	29.42	5.7	0.018	0.016
EARTH CHANNELS						
0.32	0.52	0.00067	1.04	5.0	0.059	0.077
0.34	0.58	0.0001014	0.81	3.5	0.023	0.031
0.35	0.56	0.000337	0.96	3.2	0.042	0.053
0.37	0.58	0.00064	1.08	4.1	0.055	0.071
0.40	3.90	0.000023	78.35	48.0	0.037	0.045
0.42	0.71	0.000067	2.36	7.0	0.018	0.023
0.42	0.48	0.00107	1.08	5.0	0.055	0.072
0.43	2.90	0.0000107	89.23	70.0	0.017	0.022
0.43	0.52	0.000353	1.29	5.1	0.033	0.042
0.44	0.77	0.00029	2.47	6.6	0.038	0.048
0.45	0.40	0.000919	0.76	3.9	0.043	0.055
0.54	0.14	0.00135	0.56	7.2	0.023	0.027
0.61	1.08	0.00017	7.83	11.1	0.026	0.034
0.65	0.68	0.00095	2.69	5.5	0.046	0.054
0.65	1.74	0.00028	23.40	19.6	0.043	0.055
0.67	1.04	0.000098	15.44	21.0	0.018	0.022
0.70	1.06	0.000398	9.28	11.5	0.036	0.044
0.71	0.48	0.00043	2.34	6.5	0.022	0.027
0.78	0.88	0.000175	6.37	8.4	0.019	0.023
0.81	1.80	0.000217	35.99	23.0	0.032	0.040
0.82	2.99	0.0000345	68.29	27.0	0.018	0.022
0.84	0.35	0.0013	1.57	5.0	0.027	0.032
0.84	1.32	0.00033	24.59	21.0	0.032	0.039
0.87	0.69	0.000594	5.18	8.0	0.028	0.033
0.87	1.15	0.00029	12.35	11.0	0.027	0.032
0.88	0.49	0.0014	2.47	5.5	0.034	0.039
0.91	0.88	0.000516	43.05	55.0	0.028	0.034
0.91	0.57	0.00034	3.30	6.3	0.018	0.023
0.93	0.99	0.000134	12.77	13.5	0.016	0.018
0.97	0.50	0.00046	2.70	5.5	0.018	0.021
0.99	1.27	0.000263	35.01	28.0	0.024	0.028
1.00	0.43	0.001246	3.20	7.2	0.025	0.030
1.01	0.52	0.00056	4.03	7.5	0.020	0.022
1.01	1.83	0.00012	38.55	19.5	0.020	0.024
1.03	1.07	0.000533	10.08	8.0	0.030	0.035
1.04	0.75	0.00082	6.37	7.0	0.031	0.034
1.04	0.86	0.00012	10.68	11.4	0.012	0.014
1.08	1.40	0.00015	23.55	14.2	0.018	0.021
1.09	0.83	0.00084	8.08	7.5	0.031	0.035
1.10	0.48	0.0014	13.30	24.0	0.027	0.031
1.12	0.96	0.00093	10.95	9.5	0.034	0.040
1.14	0.47	0.000617	2.70	4.5	0.017	0.020
1.14	0.50	0.00068	2.53	4.0	0.019	0.022
1.14	1.03	0.000493	15.72	12.0	0.026	0.030
1.15	0.91	0.00040	12.70	12.0	0.021	0.024
1.16	2.06	0.00026	42.35	15.4	0.029	0.034
1.17	0.95	0.00032	9.90	9.0	0.019	0.022
1.18	1.36	0.000411	19.35	10.5	0.027	0.032
1.18	1.32	0.000698	27.38	14.5	0.035	0.040
1.18	1.39	0.000695	27.15	14.0	0.036	0.042
1.19	1.52	0.00020	32.72	16.6	0.021	0.024
1.19	0.65	0.00075	4.63	5.5	0.022	0.026
1.20	0.66	0.001168	4.44	5.0	0.029	0.032
1.20	0.73	0.00163	6.97	7.2	0.036	0.041
1.23	0.83	0.000842	9.20	7.5	0.028	0.031
1.24	1.06	0.000357	15.22	10.5	0.021	0.024
1.26	1.81	0.00028	42.35	16.4	0.026	0.030
1.28	2.20	0.0000804	85.51	29.0	0.015	0.017
1.32	0.47	0.00368	2.57	3.5	0.038	0.042
1.34	1.11	0.00038	17.45	10.0	0.021	0.023
1.37	1.32	0.00091	23.64	11.5	0.035	0.040

TABLE 7.—COMPARISON OF COEFFICIENTS OF FRICTION—(*Continued*)

Measured velocity, in feet per second	Hydraulic radius, in feet	Slope, in feet per foot	Discharge, in cubic feet per second	Width at surface, in feet	Coefficient of roughness from Eqs. 33	Manning's coefficient of roughness
			EARTH CHANNELS—(*Continued*)			
1.38	0.71	0.000875	7.90	7.8	0.023	0.025
1.41	0.59	0.00176	9.52	11.0	0.027	0.031
1.44	1.06	0.000481	27.16	16.5	0.021	0.024
1.44	1.87	0.00060	72.00	23.0	0.034	0.039
1.44	1.84	0.000746	45.80	15.0	0.038	0.042
1.45	1.02	0.000812	14.28	8.5	0.026	0.030
1.45	1.80	0.000894	40.93	13.0	0.036	0.046
1.46	2.34	0.000111	92.08	25.5	0.016	0.019
1.47	1.66	0.00026	40.32	14.7	0.020	0.023
1.48	1.60	0.00027	87.29	35.8	0.020	0.023
1.51	1.69	0.00022	42.16	14.8	0.017	0.021
1.52	1.62	0.00115	31.07	10.7	0.041	0.046
1.55	1.88	0.00024	57.98	18.2	0.020	0.023
1.56	2.04	0.00036	57.98	16.1	0.020	0.029
1.57	1.57	0.000478	31.86	9.5	0.026	0.028
1.59	2.08	0.00020	95.31	27.0	0.019	0.022
1.60	1.22	0.001065	23.64	11.5	0.031	0.035
1.61	0.65	0.0016	5.66	4.4	0.026	0.028
1.61	2.04	0.00032	95.30	27.0	0.022	0.027
1.63	1.97	0.000262	92.80	27.0	0.021	0.023
1.66	1.52	0.00038	62.00	25.0	0.020	0.023
1.67	2.85	0.000127	137.89	26.5	0.019	0.020
1.68	1.52	0.00058	41.20	14.5	0.026	0.028
1.69	1.69	0.00050	100.00	34.0	0.023	0.028
1.69	0.56	0.0033	8.08	8.4	0.032	0.034
1.72	1.20	0.000831	20.12	8.5	0.027	0.028
1.73	1.88	0.000634	27.83	12.0	0.028	0.033
1.74	0.33	0.00334	15.29	17.1	0.023	0.024
1.75	1.29	0.000737	26.06	10.0	0.025	0.027
1.78	1.11	0.000736	26.35	12.0	0.023	0.024
1.82	2.89	0.000248	164.45	28.0	0.023	0.026
1.85	2.20	0.000215	112.50	26.0	0.018	0.020
1.86	1.13	0.00038	140.55	68.0	0.015	0.017
1.88	2.79	0.000152	148.66	26.5	0.017	0.020
1.90	1.93	0.000386	84.32	21.1	0.022	0.024
1.90	0.71	0.000175	9.98	6.4	0.025	0.026
1.94	1.07	0.00062	22.28	10.1	0.019	0.020
1.96	1.11	0.00049	32.27	13.6	0.017	0.018
1.97	1.62	0.00060	60.23	17.6	0.023	0.026
2.00	2.03	0.000312	112.50	26.5	0.019	0.021
2.00	2.13	0.000335	102.22	21.5	0.021	0.023
2.02	1.60	0.000273	64.02	18.9	0.015	0.017
2.08	3.69	0.000168	336.91	38.0	0.020	0.022
2.09	2.13	0.000377	83.68	15.5	0.021	0.023
2.09	1.80	0.00044	67.00	16.5	0.020	0.022
2.10	2.80	0.000187	171.60	27.0	0.018	0.020
2.14	1.74	0.00046	68.56	14.9	0.020	0.022
2.22	2.62	0.00017	310.84	51.0	0.015	0.017
2.30	2.12	0.000295	106.93	20.0	0.017	0.018
2.35	3.02	0.00038	218.33	27.0	0.021	0.026
2.36	1.86	0.00027	109.56	23.9	0.015	0.016
2.36	0.91	0.003088	14.35	5.5	0.032	0.032
2.45	2.60	0.00023	167.56	24.0	0.016	0.018
2.49	1.52	0.00077	51.36	12.0	0.021	0.022
2.50	1.01	0.002683	20.32	6.5	0.031	0.031
2.56	2.85	0.000308	207.00	24.0	0.020	0.021
2.56	2.41	0.00033	169.50	27.0	0.018	0.019
2.56	2.40	0.000154	330.00	51.0	0.011	0.013
2.58	1.20	0.00083	45.90	13.7	0.018	0.019
2.67	2.38	0.00072	171.34	24.0	0.025	0.027
2.72	2.04	0.000438	161.75	29.0	0.017	0.018
2.82	1.65	0.001105	83.86	16.5	0.024	0.025
2.93	0.55	0.00616	7.60	3.7	0.028	0.027
2.94	2.84	0.00025	372.10	48.0	0.015	0.017
3.12	2.16	0.000798	131.33	17.0	0.022	0.023
3.62	2.49	0.00031	225.55	22.9	0.013	0.013
3.86	1.42	0.00220	143.60	25.5	0.023	0.023
4.66	1.87	0.00366	380.00	43.7	0.030	0.029

TABLE 7.—COMPARISON OF COEFFICIENTS OF FRICTION—(*Continued*)

Measured velocity, in feet per second	Hydraulic radius, in feet	Slope, in feet per foot	Discharge, in cubic feet per second	Width at surface, in feet	Coefficient of roughness from Eqs. 33	Manning's coefficient of roughness
METAL FLUMES						
1.38	0.91	0.000175	8.08	5.2	0.013	0.013
1.43	0.84	0.00020	6.81	4.5	0.013	0.013
1.66	0.38	0.0005	1.71	2.1	0.010	0.010
1.68	0.31	0.0013	1.34	2.2	0.015	0.015
1.82	0.32	0.0012	1.39	2.0	0.013	0.013
1.92	1.04	0.000892	14.70	6.0	0.022	0.024
2.37	0.41	0.00113	3.36	3.1	0.012	0.012
2.55	0.41	0.00117	4.02	3.5	0.011	0.011
2.88	0.32	0.00175	2.57	2.6	0.010	0.010
4.40	0.69	0.0022	19.59	5.8	0.013	0.012
5.10	0.84	0.00411	23.72	4.2	0.018	0.017
5.34	0.61	0.0023	19.59	5.7	0.010	0.010
5.35	1.07	0.00386	39.19	5.1	0.020	0.018
5.77	0.92	0.00537	33.30	5.0	0.020	0.018
6.01	0.37	0.0052	7.04	2.8	0.010	0.009
COBBLED-BOTTOMED FLUMES						
1.02	0.20	0.01212	0.81	3.6	0.050	0.055
1.10	0.23	0.0170	1.17	4.6	0.057	0.065
1.33	0.23	0.0171	1.48	4.6	0.051	0.055
1.35	0.27	0.00991	0.85	1.8	0.044	0.046
1.59	1.81	0.000603	40.93	10.5	0.031	0.034
1.74	0.44	0.00093	12.90	16.0	0.014	0.015
1.77	1.17	0.001012	26.90	11.0	0.027	0.030
1.77	2.25	0.00037	75.66	16.2	0.025	0.028
1.82	0.65	0.00584	18.14	14.0	0.044	0.047
2.27	2.27	0.00055	142.10	27.0	0.024	0.028
3.57	3.02	0.001129	251.79	18.0	0.024	0.029

highest coefficient (0.057) was for a "small ditch, grass lined." Two channels described as having "much vegetation" resulted in a similar coefficient (0.055). Other channels, described as "bed harrowed, grassy," "sandy loam, grassy," and "silt and grass," resulted in coefficients of 0.018, 0.017, and 0.023, respectively.

A reliable formula should show the same coefficient of friction for a given roughness regardless of the velocity. It is reasonable to assume that there are channels of equal roughness scattered throughout the data shown in Table 7; that is, certain values of the coefficient should occur more than once in the table. This condition appears to be met by Eqs. 33, but Manning's coefficient appears to decrease with decreasing velocity. This is quite apparent in the data for earth channels. Furthermore, Manning's coefficient seems to amplify the coefficient for very rough channels far more than do Eqs. 33.

The dimensional equation can be written as

$$V = K \left(\frac{\sqrt{g\, d^3_c}}{r \nu} \right)^{2/15} \sqrt{2\, g\, R \sin \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

When tests on open channels include temperature measurements, it is possible to assign a value to coefficient K. When this has been accomplished, Eq. 34 can be used for hot water and other fluids. Further research is also needed to establish the exact value of the exponent of the term for the frictional resistance.

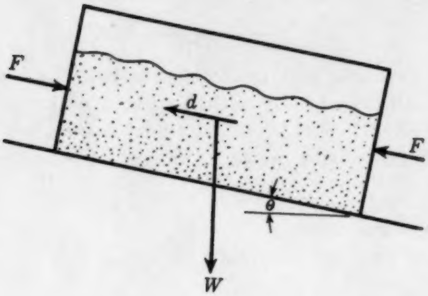

FIG. 1.—FREE-BODY DIAGRAM OF UNIFORM FLOW IN AN OPEN CHANNEL

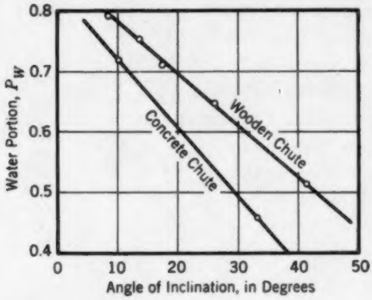

FIG. 2.—RELATION BETWEEN THE WATER PORTION AND THE ANGLE OF INCLINATION

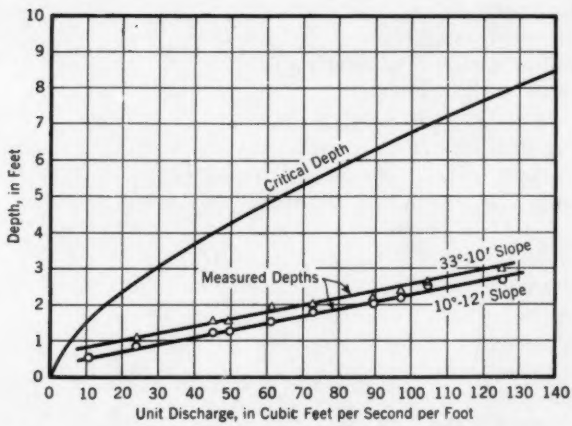

FIG. 3.—RELATION BETWEEN MEASURED AND CRITICAL DEPTHS AND THE UNIT DISCHARGE FOR THE KITTITAS WASTEWAY

APPENDIX. NOTATION

The following symbols, adopted for use in this paper and for the guidance of discussers, conform essentially with American Standard Letter Symbols for Hydraulics (ASA-Z10.2—1942), prepared by a committee of the American Standards Association, with Society representation and approved by the Association in 1942:

A = cross-sectional area;
a = radius of the outer boundary of the eddy;
B = resistance coefficient;
b = width of the channel;
C_d = coefficient of drag;
C_m = coefficient defined in Eq. 14*d*;
d = drag force;
E = specific energy;
f = Darcy's coefficient of friction;
g = acceleration of gravity;
h' = radial component of velocity;
J = a factor expressing the dimensions of the channel;
K = a constant in Eq. 28*a*;
L = length of the free-body;
M = coefficient defined in Eq. 16;
n = Manning's coefficient of roughness;
P = wetted perimeter;
P_w = water portion;
p_a = atmospheric pressure;
Q = total discharge;
Q_a = discharge of aerated flow;
Q_s = discharge of non-aerated flow;
q = discharge per unit width;
R = hydraulic radius;
R = Reynolds number;
r = radius vector of polar coordinates;
S_0 = slope of the channel;
V = mean velocity of flow;
V_t = total velocity along a streamline;
v' = circumferential component of velocity;
v'_l = local circumferential velocity at distance r'_l from the center of the eddy;
W = weight of the water in the free-body;
y = depth of flow;
γ = specific weight;
θ = angle at which the chute is set;
μ = dynamic viscosity;
ν = kinematic viscosity;
ρ = density of the fluid;
Φ = a function;
ϕ = polar coordinate; and
ψ = the stream function.